邏輯

1. Explain the following notions and theorems (30 points):
   a. Decidable set/enumerable set/effectively enumerable set
   b. The Continuum Hypothesis/The Generalized Continuum Hypothesis
   c. Semantical consequence/ syntactical consequence/logical truth/theorem
   d. The Compactness Theorem

2. Multiple choices (each question might have more than one "correct" choice) (30 points):
   a. (   ) An open sentence ((1) may be satisfied by all sequences of objects; (2) may be satisfied by no sequence of objects; (3) may be satisfied by some sequences of objects yet not satisfied by others.)
   b. (   ) A closed sentence ((1) may be satisfied by all sequences of objects; (2) may be satisfied by no sequence of objects; (3) may be satisfied by some sequences of objects yet not satisfied by others.)
   c. (   ) The set of natural numbers is a proper subset of the set of rational numbers. ((1) Therefore, there isn't; (2) Still, there is) a 1-1 onto mapping from the former to the latter.
   d. (   ) Which of the following sets of connectives is truth-functionally complete: ((1) $\{\sim, \vee\}$; (2) $\{\sim, \wedge\}$; (3) $\{\sim, \supset\}$; (4) $\{\sim, \equiv\}$; (5) $\{\supset, \equiv\}$).
   e. (   ) Which of the following statements is true: ((1) The union of a denumerable set and a finite set is denumerable; (2) The union of a countable set and a countable set is countable; (3) There are uncountably many truths of arithmetic.)
   f. (   ) Which of the following statements is true: ((1) $\aleph_0+\aleph_0 = \aleph_0$; (2) $\aleph_0\times\aleph_0 > \aleph_0$; (3) $2^{\aleph_0} = c$; (4) $\aleph_0\times c > c$; (5) $\aleph_0{}^n > \aleph_0$).
   g. (   ) Let $I(G)=\{\langle x, y\rangle \mid x\in N \text{ and } y \in N \;\&\; x\geq y\}$, then the sequence $\langle 5, 10, 15, 5, 10, 15, \ldots\rangle$ satisfies ((1) (x) Gxx; (2) (x) Gxy; (3) (x)(y)Gxy; (4) (∃x)Gxy).
   h. (   ) Although first-order logic is undecidable, there is an effective positive test for ((1) satisfiability; (2) unsatisfiability; (3) validity).
   i. (   ) If a first-order wff is satisfiable, it is satisfied in an ((1) enumerable; (2) arbitrary infinite; (3) finite) domain.
   j. (   ) Which of the following are not decidable? ((1) sentential logic; (2) monadic predicate logic; (3) first-order logic; (4) arithmetic).

3. Specify a separate model for each of the following items showing that (20 points):
   a. "(x)(y)(z){[(Rxy & Ryz) ⊃ Rxz] & ~Rxx} & ~(y)(∃x)Rxy" is satisfiable in a model with finite domain.
   b. It is false that (x)(Fx ⊃ (∃y)(Gy & Rxy)) ⊨ (∃y)(Gy & (x)(Fx ⊃ Rxy)).

4. Choose one of the followings (20 points):

   a. Prove Lindenbaum's lemma for first-order logic: If K is a consistent first-order theory, then there is a first order theory K' that is a maximal consistent extension of K with the same formulas as K.

   b. Let ϕ be any wff, x and y any variables. <D, V> any model, and μ any value assignment to the variables. Then, where σ is just like μ except that $\sigma(x) = \mu(y)$, prove that $V_\sigma(\phi) = V_\mu(\phi[y/x])$. ($\phi[y/x]$ differs from ϕ only in that wherever ϕ has free variable x, $\phi[y/x]$ has variable y which is free in $\phi[y/x]$ but not in ϕ.

   c. Prove: (a) $\Gamma \cup \{\sim\phi\}$ is first-order (syntactically) inconsistent iff $\Gamma \vdash_{FOL} \phi$; (b) $\Gamma \cup \{\phi\}$ is first-order (syntactically) inconsistent iff $\Gamma \vdash_{FOL} \sim\phi$.

   d. Prove that the set of all subsets of natural numbers is not enumerable.