國立中正大學 碩士班甄試 邏輯試題 2010/11/19

In this test,

"¬" means "not",

"∧" means "and",

"∨" means "or",

"→" means "if...then...",

"↔" means "if and only if",

"∀x" means "for all x", and

"∃x" means "for some x".

I. True or False

Please answer each of the following questions simply by writing **True** or **False. (20 points; 4 points each)**

1. [(A∧¬B)∨(B∧¬C)∨(C∧¬A)]→[(A∧B∧C∧D)→(E↔F)] is a tautology.
2. ∃x(P(x)↔R(x)) is logically equivalent to ∃xP(x)↔∃xR(x).
3. Assume that only one of the following two sentences is true: (1) Pigs can fly unless Kant is not right; (2) Kant is not right only if pigs can fly. Based on this assumption, it is true that if pigs can fly, then I will cry.
4. If P and S are consistent and S and Q are inconsistent, then P cannot imply Q.
5. Suppose that most philosophers are truth-pursuers and that most truth-pursuers are smart. Then we can conclude that most philosophers are smart.

II. Please give counterexamples to the following two **invalid** arguments. **(30 points; 15 points each)**

(1) ∃x(Px→∀yRy) /∴ ∃xPx→∀yRy

(2) ∀x¬R(x, x)∧∀x∃yR(x, y)∧∀x∀y∀z(R(x, y)→(R(y, z)→R(x, z)))

  /∴∃x∀y(x≠y→R(x, y))

III. Let "Lxy" stand for "x loves y",

   "Hxy" stand for "x hates y" and

   "Px" stand for "x is a philosopher".

Please symbolize the following sentence. **(20 points)**

There is some philosopher who hates exactly two persons who are not philosophers and who love each other but no one else.

IV. Please prove the following **valid** argument. (You may use the system on the next page, but virtually all formal proof systems are acceptable. Just make your proof as clear as possible). **(30 points)**

$\forall x \neg[(P(x) \leftrightarrow R(x)) \leftrightarrow Q(x)]$, $\exists x \exists y (\neg R(x) \vee S(x, y))$ /$\therefore$ $\exists x \exists y [Q(x) \rightarrow (\neg S(x, y) \rightarrow P(x))]$